B.C.A Third year
October 2016

Duration : 70 mins   Max marks : 50

Note: Attempt two questions each from Section A and Section B and in these sections each question carries 10 marks. Section C is Compulsory and each question carry 2 marks.

Section -A

1. Prove that (i) $(A \cup B)^c = A^c \cap B^c$ and (ii) $(A \cap B)^c = A^c \cup B^c$.
2. A class has 175 students. The following table shows that the number of students one or more of the following subjects in this class. How many students are enrolled in Mathematics alone, Physics alone, Chemistry alone ? Are these students who have not offered any of these subjects ?

Subjects	$No.of Students$
Maths	100
Physics	70
Chemistry	46
Maths and Physics	30
Maths and Chemistry	28
Physics and Chemistry	23
Maths,Physics and Chemistry	18

3. =Let $A = (2,3,5,8)$, $B = (4,6,16)$, $C = 1,4,5,7$. Let $R = ((a,b) : a/b)$ and $S = ((b,c) : b \leq c)$ be relations from A to B and B to C. Find the composite relation SoR. If M_{SoR},M_R,M_S be the adjacency matrix of SoR, R and S resp. Then show that M_{SoR}=$M_R.M_S$.
4. For $\frac{a}{b}, \frac{c}{d} \in \mathbb{Q}$ - the set of rational numbers, define $\frac{a}{b}$ R $\frac{c}{d}$ if and only if $ad = bc$. Show that R is an equivalence relation on $\mathbb{Q}$.

Section -B

1. Use principle of Mathematical Induction, to show that

$$\frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \ldots\ldots \frac{1}{(2n-1)(2n+1)} = \frac{n}{2n+1} \forall n \in \mathbb{N} \quad (1)$$

2. Use Induction, Prove $3^{2n+2} - 8n - 9$ is divisible by 64 for every natural number n.
3. Is $f(x) = \frac{x-1}{x+1}$ is invertible in its domain ?. If so, find f^{-1}.
4. Let $f : \mathbb{R}\times\mathbb{R} \longrightarrow \mathbb{R}\times\mathbb{R}$ be defined by $f(x,y) = (x+2y, y-x)$. Let $g : \mathbb{R}\times\mathbb{R} \longrightarrow \mathbb{R}\times\mathbb{R}$ be defined by $g(t) = (3t, t^2)$. Let $h(x,y) = x + 2y$. Find fog, hof, $ho(fog)$.

Section -C

1. Let $A = (1,2,3)$, $B = (1,2)$, $C = (2,3)$. Find minsets generated by B and C.
2. Let R be a relation on set $A = (1,2,3)$ defined by $R = ((1,1),(1,2),(2,3)$. Find symmetric closure of R.
3. If $P(n)$ denotes statement $2^n \geq 3n$. Show that if $P(n)$ is true, then $P(n+1)$ is true.
4. Given $f(x+1) = 3x+5$, Evaluate $f(2x)$.
5. How many relations are possible from a set A of m elements to another set B of n elements ?

OCTOBER EXAMINATION – 2012 (DISCRETE MATHS)

Class : B.C.A. – III

SUBJECT : DISCRETE MATHEMATICS

TOTAL MARKS : 40 **TIMING : 01:30 min.**

Note : Six questions in all Selecting 2, 2 and 1 from section A, B & C respectively. Section D is compulsory.

Section – A

1. By finding generating function of sequences S(n), find solution of recurrence relation S(n + 2) – 7S(n + 1) + 12S(n) = 0 for $n \geq 0$ given S(0) = 2, S(1) = 5.
2. Solve the relation $a_n - a_{n-1} - 2a_{n-2} = 3n.4^n, n \geq 2$
3. If (G(z) is generating function of numeric function S_n then show generating function of $\alpha^n S_n$ $G(\alpha z)$.
   Use the result to find generating functions sequences
   (i) $n5^n$
   (ii) $1, \frac{2}{3}, \frac{3}{9}, \frac{4}{27}, ...$

SECTION – B

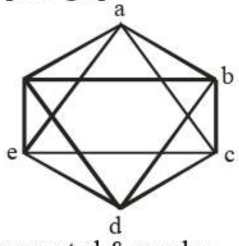
4. For any two sets A & B prove that
   $A \cup B = A \cap B$ iff A = B
5. Let A = {1, 2, 3, 4), B_1 = {1, 2}, B_2 = {2, 3, 4}
   (a) Find the minsets and maxsets generated by B_1 & B_2
   (b) Write B_2^c as minsets normal form. What is maxset form of B_2^c ?
   (c) Do minsets form a partition of A? Check.
6. A B.A. III class has 175 students. The following table shows the numbers of students studying one or more of the following subject in this class:

Subject	No. of students
Maths	100
Economics	70
Pol. Sc.	40
Maths & Eco	30
Maths & Pol. Sc.	28
Eco & Pol. Sc.	23
Maths, Eco & Pol. Sc.	18

How many students are enrolled in Maths alone, Economics alone and Pol. Sc. alone? Are students who have not offered any of these three subjects?

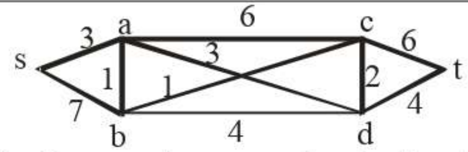
SECTION – C

7. (a) Given graph
   Justify following statements
   (i) It is a complete graph



   (ii) Graph is connected & regular
   (iii) It is a planar graph? If true find number of regions using Euler's formula.
   (iv) Graph is Eulerian.

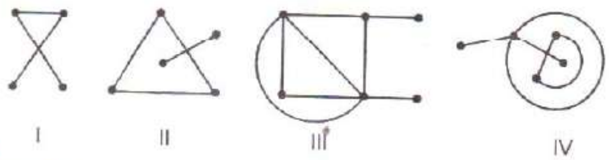
(b) Find shortest path using Dijkstra's algorithm from s to t.



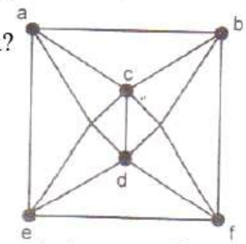
(c) (i) Draw graph corresponding to the adjacency matrix

$$A = \begin{bmatrix} 1 & 3 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix}$$

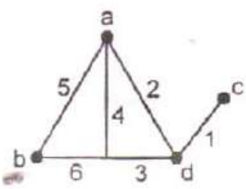
(ii) Consider the multigraph which of them are connected, loop free & simple graph.

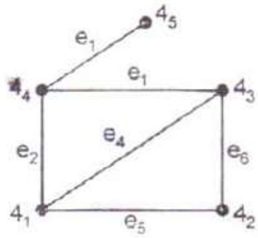


8. (a) (i) Does the graph Euler's path? If so, write one.
   (ii) Is the graph Hamiltonian?
   (iii) Is the graph bipartite?



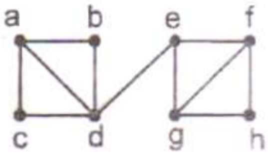
   (b) Let G = < V, E > be a connected planar graph and let R the number of region defined by any planar depiction of G. Then we have
   R = |E| - |V| + 2
   (c) (i) A graph G has 21 edges, 3 vertices of degree 4 and other vertices are of degree 3. Find the number of vertices in G.
   (ii) Show that the graphs G & G' are isomorphic



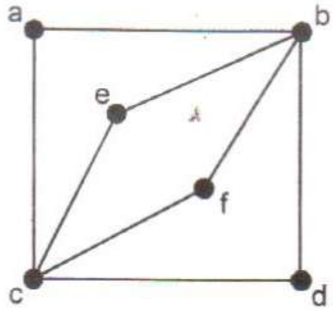


9. (a) Write the generating functions of the sequence $S_n = 4.3^n + 5(-1)^n + 9$
   (b) If A & B are two subsets of a universal set then $A - B = A \cap \overline{B}$.
   (c) (i) Find complement $\overline{G}$ of the graph G.

   (ii) Find cut edge & cut vertex.



(d) Define Hamiltonian graph. Is the graph given below a Hamiltonian? Justify your answer.



(e) Is there a simple graph G with the degree sequence (1, 1, 3,4, 6, 7)?

House Examination – 2016

Class : B.C.A III (27)

SUBJECT : DISCRETE MATHS

Time : 2 hrs. **M.M. : 40**

Note : Attempt 5 questions selecting at least 2 from each Section. Question No. IX is compulsory.

SECTION – A

1. (a) If $f(x) = \frac{1}{1-x}$. What is f[f{f(x)}]? Find domain of f(x).

   (b) Is $f(x) = \frac{x-1}{x+1}$ invertible in its domain? If so find f^{-1}. 4, 4

2. Let A = B = {1, 2, 3, 4, 5}. Define function f : A $\rightarrow$ B such that
   (i) f is one to one and onto.
   (ii) f is neither one to one nor onto
   (iii) f is one-one but not onto
   (iv) f is onto but not one to one 8

3. Define composition of two functions. Let f and g be two functions from R $\rightarrow$ R defined by $f(x) = x^2 + 3x + 2$ and g(x) = 4x – 1. Find fog and gof. Also calculate (gof) (-1) and (fog) (-1). Is composition commutative? 8

SECTION - B

4. (a) For a function g : R $\rightarrow$ R, determine whether the following functions are one-to-one and onto. If the function is not onto, determine range g(R).
   (i) g(x) = x + 7 (ii) $g(x) = x^2 + x$. 8

5. If f, g : R $\rightarrow$ R are defined respectively by
   $f(x) = x^2 + 3x + 1$, g(x) = 2x – 3
   find formulae for (i)f o g (ii) gof (iii) fof (iv) gog. 8

6. Let f and g be functions from R to R defined by f(x) = [x] and g(x) = |x|. Determine whether fog = gof. Illustrate will numeric example. 8

SECTION – C COMPULSORY

7. Let f : Z $\rightarrow$ Z defined by $f(x) = x^2$. Prove that f is not one-one 2

8. Under that condition a constant function can be (i) one-to-one (ii) an onto function. 2

9. Let A = {1, 2, 3}. Define f : A $\rightarrow$ A by f(1) = 2, f(2) = 1, f(3) = 3. Find f^2, f^4. 2

10. Explain Inverse mapping with example 2

December Examination – 2009

Class : B.C.A III (27)

SUBJECT : MATHEMATICS

Time : 3 hrs. **M.M. : 100**

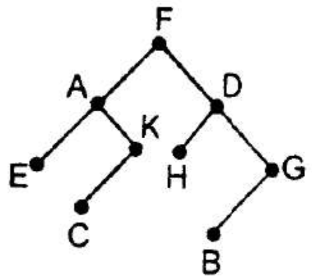
Note : Attempt 5 questions selecting at least 2 from each Section. Question No. IX is compulsory.

SECTION – A

1. (a) Is A and B are two sets such that n(A – B) = 24 + x n (B – A) = 2x and n(A ∩ B) = 2x Illustrate the information by Venn's diagram. If n(A) = n(B) find (i) numerical value of x (ii) n (A ∪ B).
   (b) If A and B two subsets of a universal set, then show that $A - B = A \cap \overline{B}$
   (c) Let A and B be two sets. Prove that A – B = A ∩ B^C
2. (a) If A, B, C are any three sets, show that (A ∩ B) × C = (A × C) ∩ (B × C)
   (b) If R is the relation in N × N differed by (a, b) R (c, d) iff a + d = b + c. Is the relation R an equivalence relation?
   (c) If R = {(a, b)|a – b is even} are two relations on A] {1, 2, 3, 4} then
   (i) find matrices of R and S
   (ii) Draw diagraphs of R and S.
   (iii) Using matrices of R and S find the relation RS & SR
   (iv) Show that $R^2 = S^2$
   (b) If $f(x) = \frac{1}{1-x}$. What is f[f{f(x)}]? Find domain of f(x).
4. (a) Is $f(x) = \frac{x-1}{x+1}$ invertible in its domain? If so find f^1.
   (b) Let A = B = {1, 2, 3, 4, 5}. Define function f : A → B such that
   (i) f is one to one and onto.
   (ii) f is neither one to one nor onto
   (iii) f is one-one but not onto
   (iv) f is onto but not one to one

SECTION - B

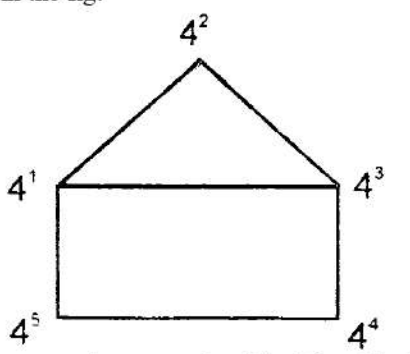
5. (a) Solve the recurrence relation $a_k = 8a_{k-1} + 10K - 1$ with initial condition a_0 = 1.
   (b) Solve S(n) + 3S(n – 1) – 4S(n – 2) = 0, n ≥ 2 S(0) = 3, S(1) = - 2 using generating function.
6. (a) Prove that in a graph the number of vertices of odd degree is even.
   (b) Determine whether or not each of the following graphs G(V, E) in a graph where V = {A, B, C, D}
   (i) E = {(A, B), (A, C), (A, D), (B, C), (C, D)}
   (ii) E = {(A, B), (B, B), (A, D)
7. (a) From expression tree for ((2 × x) + y) × ((5 × a) – b)^3.
   (b) Draw ordered tree for (2a + b) (5x – y)^2
8. (a) Find the pre-order, In order and post order traversal of the tree given in the fig.



(b) (i) Place the addresses in lexicographic order i.e. draw the corresponding ordered rooted tree, for 2.2.1, 1, 3.2, 1.1.1, 1.1.2, 2.2.1.1, 2.1, 0, 3.2.1, 3.1, 3, 2.1.1, 2.2, 1.1, 3.2.1.1, 2, 3.2, 1.2, 1.1.2,

SECTION – C

9. (i) What is the set {x|x ∈ R, x^2 = 4, x^2 – 3x + 2 = 0}
(ii) How many relations are possible from a set A of m elements to another set B of n element? Why?
(iii) Find the relation R if $M_R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$.
(iv) Write all possible relations from A = {0} to B = {1, 2}
(v) Is the function g(n) = x^2 + x one-one
(vi) Solve $s_n - 4s_{n-1} + 3s_{n-2} = 0$
(vii) If V = {a, b, c, d} and E = {(a, b), (a, d), (b, c), (b, d), (c, d)} be the vertex set and edge set of a graph. Is it a simple graph?
(viii) Is there a graph with 8 vertices of deg. 2, 2, 3, 6, 5, 7, 8, 4. Justify your answer?
(ix) Write the adjacency matrix of the simple graph G given in the fig.



(x) Represent the expression (A + B) × (C – D) as a Binary tree and write prefix form of expression.

BCA – III (P.U)
DISCRETE MATHEMATICS (TEST PAPER)
CODE BCA (27)

Max. Marks – 90

Note (i) Attempt one question from each section carrying 18 marks each.
(ii) Section-E is compulsory. (9 questions carrying two marks each)

Section – A

Ques.1(a) Let B_1, B_2, B_3 be the subsets of the universal set U.
(i) Find all the minsets generated by B_1, B_2, B_3
(ii) Illustrate via. a venn diagram, all the minsets obtained in part (i).
(iii) Express the given sets B_1, $B_1 \cap B_2$ in minset normal form

(b) Consider R = {(a, b): |a - b| = 1}
and S = {(a, b): a – b is even } are two relations on A = {1, 2, 3, 4}. Then
(i) Find the matrices of R and S.
(ii) Draw the diagraphs of R and S.
(iii) Using the matrices of R and S. Find the relation RoS.
(iv) Show that $R^2 = S^2$.

(c) A function f : X → Y is invertible iff f is 1-1 and onto.

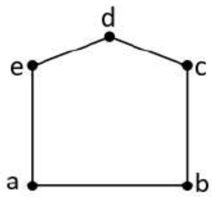
Ques. 2 (a) Solve $S_n - 4\,S_{n-1} + 4S_{n-2} = 3n + 2^n$, with $S_0 = S_1 = 1$
(b) If $S_n + 3S_{n-1} - 4\,S_{n-2} = 0$ with $S_0 = 3$, $S_1 = -2$
Find the generating function
(i) Using the definition of generating function.
(ii) Write down the sequence satisfying it.

Section – B

Ques. 3. (a) Prove that the number of edges in a complete graph with n vertices is $\frac{n(n-1)}{2}$.
(b) Can a graph with 7 vertices be isomorphic to its complement? Justify.

Ques. 4 (a) Prove that the following statements are equivalent for a graph G.
(i) G is 2-colorable.
(ii) G is bipartite.
(iii) G contains no odd cycle.

(b) For a given graph:
(i) Write down the adjacency list.
(ii) Find the adjacency matrix.
(iii) Find the incidence matrix.
(iv) Draw the compliment of the graph.



Section – C

Ques. 5 (a) Describe a FSM of addition modulo 3 of positive integers. If A, B, C denotes the states that the modulo 3 sum of all digits in input is 0, 1, 2, respectively. Also, represent the FSM graphically.
(b) Prove that $7x^2$ is $O(x^3)$. Is it true that x^3 is $O(7x^2)$

Ques. 6 (a) Show that a lattice with 3 or fewer elements is a chain.
(b) For any positive integer m, Let D_m denotes the set of all divisors of m ordered by divisibility. Draw the Hasse diagram of D_{60}.

Section – D

Ques.7 (a) Give the Disjunctive Normal Form of the Boolean Function $f(x, y, z) = (x \vee y) \wedge (x \vee y') \wedge (x' \wedge z)$
(b) Draw the circuit diagram of the Boolean Function $f(x, y, z) = x'\,y'\,z + x'\,y\,z + x.\,y'$

Ques. 8 (a) Test the validity of the argument:

If the wages increase, then there will be inflation, the cost of living will not increase if there is no inflation. Wages will increase. Therefore, the cost of living will increase.

(b) $7^{2n} + 2^{3n-3}.3^{n-1}$ is divisible by 25, $n \in N$

Section – E

Ques. 9. I. For any two sets A and B. Prove that $n(A \times B) = n(A).n(B)$

II. If $X = \{1, 2, 3, 4, 5\}$ and $R = \{(1, 1) (1, 2) (2, 3) (3, 5) (3, 4) (4, 5)\}$

Determine (i) R^2

(ii) R^∞

III. If $f(x) = x^2 + 1$ and $g(x) = 3x + 1$

Then describe the following functions.

(i) fog(x) (ii) fof(x)

IV. Find the sequence whose generating function is $G(S, z) = \frac{1}{1-z-z^2}$

V. Which of the following multigraphs are connected?

(i) (ii) (iii)

VI. Define coloring of a graph and its chromatic number. Also give the chromatic number of the given graph

VII. Consider C(x) : x is a cold blooded

F(x) : x is a fish.

S(x) : x lives in sea.

Translate info formula : Every fish is cold blooded.

Translate info English : $(\exists x)(S(x) \wedge F(x))$

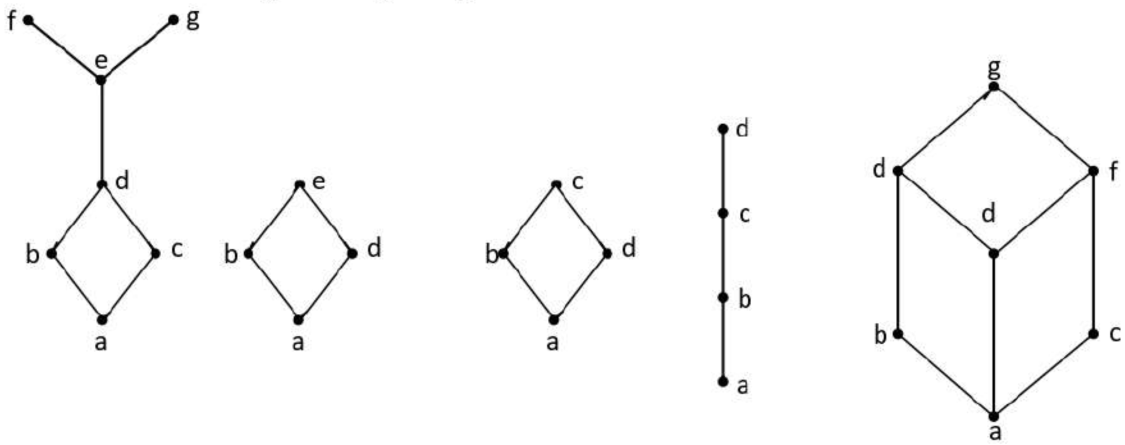
VIII. Identify the supremum, Infimum, wherever they exist for $(R, \leq)$ where '$\leq$' has its usual meaning.

(i) $A = \{x \in R : x \geq 0\}$

(ii) $A = \{(a, b): x \in R, a < x < b\}$

(iii) $A = \{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4} \ldots\ldots\ldots\}$

IX. Which of the following Hasse diagram represents a lattice.



BCA – III (P.U)
DISCRETE MATHEMATICS (TEST PAPER)
CODE BCA (27)

Max. Marks – 90

Note (i) Attempt one question from each section carrying 18 marks each.
(ii) Section-E is compulsory. (9 questions carrying 2 marks each)

Section – A

Ques. (a) A class has 175 students. The following table shows that the number of students studying one or more of the following subjects in the class.

Subjects	No. of Students
Mathematics	70
Physics	46
Chemistry	30
Maths and Chemistry	28
Physics and Chemistry	23
Maths, Physics and chemistry	18

How many students are enrolled in Maths alone, Physics alone, Chemistry alone? Are there any students who have not offered any of these subjects.

(b) Let A = {1, 2, 3, 4} and a relation on A as "R = {(a, b) : |a - b| = 2} Evaluate the Transitive closure of R.

(c) Is $f(x) = \frac{x-1}{x+1}$ invertible in its domain? If so, find f^{-1}.

Ques. 2 (a) Solve $S_n - 4S_{n-1} + 3S_{n-2} = n^2$.
(b) $S_n - 7S_{n-1} + 12S_{n-2} = 0$, where $S_0 = 2$, $S_1 = 5$.
Find the generating function.
(i) Using definition of generating function.
(ii) Write down the sequence satisfying it.

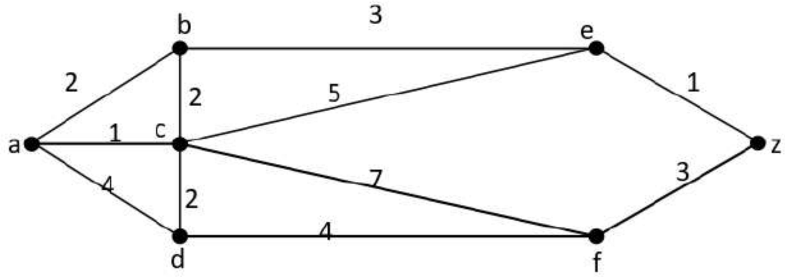
Section – B

Ques. 3. (a) State and prove the Euler's Formula.
(b) A machine operator processes 5 types of items on his machine each week and must choose a sequence for them. The set up cost per change depends on the items present on the machine and set-up to be made according to the following table:-

To item

From item		A	B	C	D	E
	A	∞	40	70	30	40
	B	40	∞	60	30	40
	C	70	60	∞	70	50
	D	30	30	70	∞	70
	E	40	40	50	70	∞

Ques.4 (a) Let G = <V, E> be a simple planar connected graph with more than one edge then the following inequalities hold:
(i) 2 |E| > 3 R
(ii) |E| ≤ 3|V| - 6.
(iii) There is a vertex v in G s that deg (v) ≤ 5
(b) Give the shortest path between a and z.



SECTION – C

Ques.5 (a) Let w = abcde. Find all the subwords of w. Which of them are initial segments.

(b) Prove that $f(x) = 8x^3 + 5x^2 + 7$ is $\Omega(g(x))$. Where $g(x) = x^3$.

Ques.6 (a) Let $(L, \leq)$ be a lattice, for any element $a, b \in L$. Prove that $a \wedge b = a \Leftrightarrow a \vee b = b$

(b) State and Prove the uniqueness theorem for the compliment of an element of a lattice.

SECTION – D

Ques. 7 (a) Reduce the expression using the rules of Boolean Algebra

$\overline{A.\overline{B} + \overline{A.B.C} + A.\left(B + A.\overline{B}\right)}$

(b) Using Boolean Algegra, Prove that

(i) abc + abc' + ab'c + a'bc = ab + bc + ca.

(ii) $xy + xz + yz = xy + (x \oplus y)z$

Ques.8 (a) Show that $(\sim p \wedge \sim q) \rightarrow (p \rightarrow q)$ is a tautology

(b) Prove by using the principle of mathematical Induction.

$1 + 2 + 3 + 4 + \ldots\ldots\ldots\ldots\ldots n < \frac{(2n+1)^2}{8}$

SECTION - E

Ques9. I. Define Partitions with an example.

II. Is the function $f(x) : R \rightarrow R$ defined as $f(x) = \frac{1}{x}$ invertible? Justify.

III. Write down the Telescoping form of

$f(x) = 2x^4 + 3x^3 + 5x^2 + 8x - 3$.

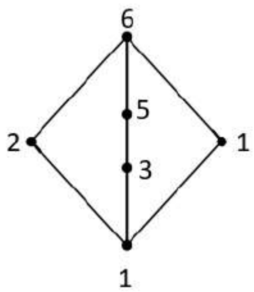
IV. Does there exist a graph with 6 vertices with degree equal to 3, 2, 4, 1, 3 and 2 respectively

V. Define a Bipartite graph and draw the complete bipartite graph $K_{3,3}$

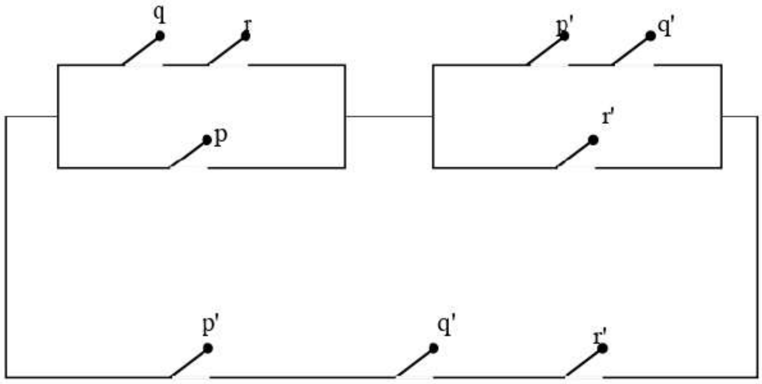
VI Define a planar graph

VII. Define atom and antiatoms of a lattice

Also give the atom and antiatoms of the given figure



VIII. Using the Boolean Algebra, Simplifying the switching circuit in the figure given below



IX. On the universe of the positive integers

$p(n) : 4n^2 - 3n = 0$

$q(n) : n^2$ is a perfect square and $n < 90$

Evaluate T_p, T_q, $T_{\sim p}$ and $T_{p \wedge q}$